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Controlling nonstationary vibrations of a plate by means of additional loads

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Abstract

The paper presents the solution of the problem related to controlling nonstationary vibrations in a certain point of a rectangular plate by introducing an additional (control) load whose variation vs. time law is to be defined. The problem is solved by using the nonclassical theory of plates and Tikhonov's regularization method.

Keywords: Inverse problem; Vibration control problem; Nonstationary loading; Rectangular plate; First kind Volterra integral equation; Regularization algorithm

1. Introduction

Developing the theoretical basics of controlling strain at nonstationary vibration of mechanical systems with distributed parameters relates to complex, and often to ill-posed inverse problems in mathematical physics. In dealing with these problems, relevant partial differential equations have to be used, the time variable therein playing a key role.

In the paper (Kozdoba and Krukovsky, 1982), it was noted that, in principle, the inverse problem can be treated as a partial case of the control problem in the presence of a control objective. Therefore, the assumptions on the ill-posedness of inverse problems in mathematical physics, including also problems in impulse strain of mechanical systems, are also true, at certain generalization, for nonstationary strain processes control problems.

At present, the methods of solving inverse problems in identifying external actions on mechanical systems with distributed parameters are developing rapidly. The baseline data for solving such problems, when establishing the causal characteristics, are taken to be quantities that can be measured as indirect manifestations (displacements and strain). Several solutions are known for nonstationary inverse problems in

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rod systems (Gladwell, 1984; Lukianova, 1985; Romanenko et al., 1989; and Krasnobaev and Potietiun'ko, 1989). The solutions to ill-posed inverse nonstationary problems in elasticity theory for rectangular plates loaded transversely as well as tangentially to the outer face surface, and to a flat spherical shell are given in papers (Yanyutin and Voropay, 2002 and Yanyutin et al., 2003). The identification of a dynamic distributed load applied non-axisymmetrically to a cylindrical shell has been described in the paper (Yanyutin and Yanchevsky, 2001). The same paper also presents the technique of controlling axial vibration processes in a certain point of a finite-length rod, and a certain area of the rod whose dimensions are small as compared to the rod length. The monograph (Skopetsky et al., 2002) also relates to research in this area.

When solving problems related to vibration of rectangular plates, expansion of sought-for quantities to functional series satisfying the boundary conditions exactly is widely used. Let us mention one such paper (Shupikov and Smetankina, 2001), which deals with the solution of a direct nonstationary problem in the theory of elasticity for a plate with an arbitrary contour. The paper considers a rectangular plate containing a plate of required shape. At this, by introducing additional loads, one can realize the procedure of satisfying the boundary conditions on its contour by controlling the strained state of the rectangular plate.

2. Direct problem

This paper, which describes the solution of the problem related to controlling the vibration (strain) process in a certain point on the median plane of a hinged rectangular plate, uses the Fourier series theory. The plate is subject to action of a transverse concentrated force $P(t)$ in point x_0, y_0 (Fig. 1), the rectangular plate being referenced to the Cartesian rectangular coordinates x, y, z. The strain process in a certain point x_s, y_s is controlled by introducing an additional concentrated force $G(t)$ applied to point x_c, y_c. The plate dimensions along the X and Y-axes are equal to l and m respectively. Since the problem stated relates to the class of ill-posed problems in mathematical physics, and it has no exact solution, Tikhonov's regularization method is used to find an approximate solution.

Let us assume that an isotropic plate with constant thickness h is subject to an impulse load in a certain point, the load's time-dependent law of variation $P(t)$ being known. The plate material is characterized by the following constants: E is Young's module and v is Poisson's ratio. An additional condition is imposed on the elastic strain of the plate, viz. the control criterion (for instance, absence of normal displacement in a certain point, or defining variation of this displacement to a specific law). To implement the required

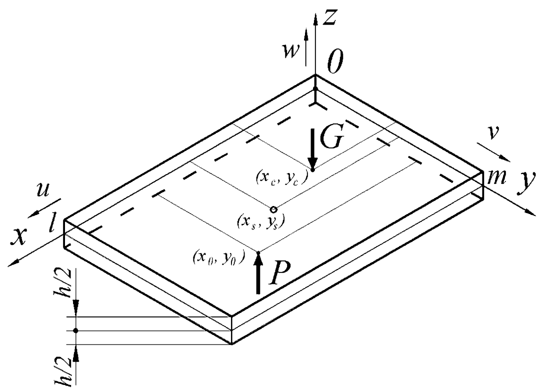

Fig. 1. Scheme for controlling nonstationary vibrations of a plate.

condition, a control load whose time-dependent law of variation $G(t)$ is to be determined is applied to the plate in a certain point. Determining this function is the matter of solving this control problem.

The system of equations for forced vibration of a rectangular isotropic elastic plate affected by two transverse concentrated loads applied to different points of the plate can be written in the form (Mindlin, 1951)

$$\begin{cases} G'h(\nabla^2 w_0 + \psi_{xy}) = \rho h \frac{\partial^2 w_0}{\partial t^2} - P_z(x,y,t) + G_z(x,y,t); \\ D\nabla^2 \psi_{xy} - G'h(\psi_{xy} + \nabla^2 w_0) = \rho \cdot I \frac{\partial^2 \psi_{xy}}{\partial t^2}; \\ \frac{D}{2}\left[(1-\nu)\nabla^2 \varphi_{xy} + (1+\nu)\nabla_1^2 \psi_{xy}\right] - G'h(\varphi_{xy} + \nabla_1^2 w_0) = \rho \cdot I \frac{\partial^2 \varphi_{xy}}{\partial t^2}, \end{cases} \tag{1}$$

where the loads are introduced by $P_z(x,y,t) = \delta(x-x_0)\delta(y-y_0)P(t)$ and $G_z(x,y,t) = \delta(x-x_c)\delta(y-y_c)G(t)$; here $P(t)$ is the specified function, and $G(t)$ is the one to be determined; and $\delta(x)$ is the delta-function. Besides, in (1) we denote $G' = k'G$; $I = h^3/12$; $D = \frac{Eh^3}{12(1-\nu^2)}$; $\psi_{xy} = \frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y}$; $\varphi_{xy} = \frac{\partial \psi_x}{\partial x} - \frac{\partial \psi_y}{\partial y}$; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$; $\nabla_1^2 = \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}$; $w_0(x,y,t)$ is the plate deflection, and $\psi_x(x,y,t)$, $\psi_y(x,y,t)$ are the angles of rotation of the normal to the median plane of the plate in the plane zOx and zOy respectively.

By solving the system of equations (1) for a hinged plate at zero initial conditions, the normal displacement of the plate's median plane and the angles of rotation of normals in a point with coordinates x_s, y_s can be found from the following relationships:

$$\begin{aligned} w(x_s,y_s,t) &= \int_0^t P(\tau)K_P^W(t-\tau)\,d\tau - \int_0^t G(\tau)K_G^W(t-\tau)\,d\tau; \\ \psi_x(x_s,y_s,t) &= \int_0^t P(\tau)K_P^{\Psi X}(t-\tau)\,d\tau - \int_0^t G(\tau)K_G^{\Psi X}(t-\tau)\,d\tau; \\ \psi_y(x_s,y_s,t) &= \int_0^t P(\tau)K_P^{\Psi Y}(t-\tau)\,d\tau - \int_0^t G(\tau)K_G^{\Psi Y}(t-\tau)\,d\tau, \end{aligned} \tag{2}$$

where

$$K_P^W(t) = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty} \frac{C_{kn}^P}{\Delta_{kn}} \sin\frac{k\pi \cdot x_s}{l} \sin\frac{n\pi \cdot y_s}{m}[\Omega_{1kn}\sin\omega_{1kn}t - \Omega_{2kn}\sin\omega_{2kn}t];$$

$$K_G^W(t) = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty} \frac{C_{kn}^G}{\Delta_{kn}} \sin\frac{k\pi \cdot x_s}{l} \sin\frac{n\pi \cdot y_s}{m}[\Omega_{1kn}\sin\omega_{1kn}t - \Omega_{2kn}\sin\omega_{2kn}t];$$

$$K_P^{\Psi X}(t) = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty} \frac{C_{kn}^P \cdot b \cdot \lambda_k^*}{\Delta_{kn}} \cos\frac{k\pi \cdot x_s}{l} \cdot \sin\frac{n\pi \cdot y_s}{m}\left[\frac{\sin\omega_{1kn}t}{\omega_{1kn}} - \frac{\sin\omega_{2kn}t}{\omega_{2kn}}\right];$$

$$K_G^{\Psi X}(t) = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty} \frac{C_{kn}^G \cdot b \cdot \lambda_k^*}{\Delta_{kn}} \cos\frac{k\pi \cdot x_s}{l} \cdot \sin\frac{n\pi \cdot y_s}{m}\left[\frac{\sin\omega_{1kn}t}{\omega_{1kn}} - \frac{\sin\omega_{2kn}t}{\omega_{2kn}}\right];$$

$$K_P^{\Psi Y}(t) = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty} \frac{C_{kn}^P \cdot b \cdot \mu_n^*}{\Delta_{kn}} \sin\frac{k\pi \cdot x_s}{l} \cdot \cos\frac{n\pi \cdot y_s}{m}\left[\frac{\sin\omega_{1kn}t}{\omega_{1kn}} - \frac{\sin\omega_{2kn}t}{\omega_{2kn}}\right];$$

$$K_G^{\Psi Y}(t) = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty} \frac{C_{kn}^G \cdot b \cdot \mu_n^*}{\Delta_{kn}} \sin\frac{k\pi \cdot x_s}{l} \cdot \cos\frac{n\pi \cdot y_s}{m}\left[\frac{\sin\omega_{1kn}t}{\omega_{1kn}} - \frac{\sin\omega_{2kn}t}{\omega_{2kn}}\right].$$

Here

$$a = \frac{G'}{\rho}; \quad b = \frac{G'h}{\rho \cdot I}; \quad d = \frac{D}{\rho \cdot I}; \quad \lambda_{kn}^2 = \pi^2\left(\frac{k^2}{l^2} + \frac{n^2}{m^2}\right); \quad \mu_{kn}^2 = \pi^2\left(\frac{k^2}{l^2} - \frac{n^2}{m^2}\right);$$

$$\lambda_k^* = \pi\frac{k}{l}; \quad \mu_n^* = \pi\frac{n}{m}; \quad \Delta_{kn} = \sqrt{\left(\lambda_{kn}^2(a+d)+b\right)^2 - 4\cdot a\cdot d\cdot \lambda_{kn}^4};$$

$$\Omega_{1kn} = \omega_{1kn} - \frac{d\cdot\lambda_{kn}^2 + b}{\omega_{1kn}}; \quad \Omega_{2kn} = \omega_{2kn} - \frac{d\cdot\lambda_{kn}^2 + b}{\omega_{2kn}}; \quad \omega_{1kn} = \sqrt{0.5\left[\left(\lambda_{kn}^2(a+d)+b\right) + \Delta_{kn}\right]};$$

$$\omega_{2kn} = \sqrt{0.5\left[\left(\lambda_{kn}^2(a+d)+b\right) - \Delta_{kn}\right]}.$$

Coefficients C_{kn}^P and C_{kn}^G in (2) represent the two-dimensional planar configuration of external loads affecting the plate. For concentrated plate loading, these coefficients are equal to

$$C_{kn}^P = \frac{1}{\rho h}\cdot\frac{4}{l\cdot m}\sin\frac{k\pi\cdot x_0}{l}\sin\frac{n\pi\cdot y_0}{m}; \quad C_{kn}^G = \frac{1}{\rho h}\cdot\frac{4}{l\cdot m}\sin\frac{k\pi\cdot x_c}{l}\sin\frac{n\pi\cdot y_c}{m}.$$

3. Inverse problem

If it is required, for instance, to eliminate the normal displacement of the plate in a point with coordinates x_s, y_s, i.e. to meet condition $w(x_s, y_s, t) = 0$, the following relationship should be fulfilled

$$\int_0^t P(\tau)K_P^W(t-\tau)\,d\tau = \int_0^t G(\tau)K_G^W(t-\tau)\,d\tau, \tag{3}$$

which, at known function $P(t)$ and sought-for function $G(t)$, is the linear Volterra integral equation of the Ist kind. Due to the "essential" ill-posedness of the respective problem, it is impossible to obtain an exact solution of Eq. (3) for an arbitrary point of application of the control action. However, it is possible to build an approximate solution by using Tikhonov's regularizing algorithm described, e.g. in (Tikhonov et al., 1990). For this, we shall write Eq. (3) in the operator form:

$$A_P \cdot p = A_G \cdot g, \tag{4}$$

where A_P is an integral operator corresponding to kernel $K_P(t-\tau)$; $A_G - K_G(t-\tau)$; p corresponds to known force $P(t)$; and g corresponds to sought-for control action $G(t)$.

By using the regularizing algorithm (Tikhonov et al., 1990), the integral equation is reduced to a regularized system of linear algebraic equations (SLAE). In matrix form, the regularized SLAE, which yields the approximate solution of Eq. (4), can be written as

$$\left(\mathbf{A}_G^T\mathbf{A}_G + \alpha\mathbf{C}\right)\cdot\mathbf{g} = \mathbf{A}_G^T\mathbf{A}_P\cdot\mathbf{p}, \tag{5}$$

where $\mathbf{A}_G$ is a matrix corresponding to integral operator A_G, whose elements are found as $a_{Gji} = K_G[(j-i)\cdot\Delta t]$; the elements of matrix $\mathbf{A}_P - a_{Pji} = K_P[(j-i)\cdot\Delta t]$; vector $\mathbf{p}$ corresponds to p, vector $\mathbf{g}$ corresponds to g; $\alpha > 0$ is the regularization parameter; Δt is the time step; J is the number of steps; and $\mathbf{C}$ is a symmetric tridiagonal $(J-1\times J-1)$ matrix of the form

$$C = \begin{bmatrix} 1+\frac{1}{\Delta t^2} & -\frac{1}{\Delta t^2} & \dots & 0 & 0 \\ -\frac{1}{\Delta t^2} & 1+\frac{2}{\Delta t^2} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1+\frac{2}{\Delta t^2} & -\frac{1}{\Delta t^2} \\ 0 & 0 & \dots & -\frac{1}{\Delta t^2} & 1+\frac{1}{\Delta t^2} \end{bmatrix}.$$

The change of control force $G(t)$ is found from the solution of SLAE (5)

$$\mathbf{g} = \left(\mathbf{A}_G^{\mathrm{T}}\mathbf{A}_G + \alpha\mathbf{C}\right)^{-1}\mathbf{A}_G^{\mathrm{T}}\mathbf{A}_P \cdot \mathbf{p}. \quad (6)$$

Relationship (6) is the final expression for solving the control problem.

4. Numerical results

Examples of computations for damping vibrations in a point of a plate with coordinates x_s, y_s are given. The following parameters were taken for computation: $\rho = 7890$ kg/m^3; $E = 2.07 \cdot 10^{11}$ Pa; $\nu = 0.3$; $h = 0.04$ m; $l = 0.6$ m, $m = 0.4$ m; the coordinates of applying the disturbing force are $x_0 = 0.45$ m and $y_0 = 0.3$ m; the control point coordinates are $x_s = 0.3$ m and $y_s = 0.2$ m; and the coordinates of applying the control force are $x_c = 0.2$ m and $y_c = 0.2$ m. The number of terms in the respective double Fourier series was taken to be $50 \cdot 50$.

We considered plate loading with an infinite impulse: $P(t) = q \cdot H(t)$, the intensity of the basic action force being $q = 10^5$ N. The results of computing the control force are shown in Fig. 2a, where curve 1 corresponds to the disturbing force vs. time law of variation, and curve 2 corresponds to the control force. We investigated the possibility of controlling the vibrations with a "simplified" force whose values were equal to the mean arithmetic values corresponding to the maximums and minimums of curve 2 in Fig. 2a—line 3 (Fig. 2a). Fig. 2b shows the deflections in the control point for the following three cases: curve 1 is absence of control ($G(t) = 0$); curve 2 corresponds to control at $G(t)$ related to curve 2 in Fig. 2a; and curve 3 corresponds to action of the "simplified" control force (curve 3 in Fig. 2a).

Fig. 3 shows the distribution of deflection over the plate when controlling the vibrations to function $G(t)$, which corresponds to curve 2 in Fig. 2a at time points $t = 2 \cdot 10^{-5}$ s (Fig. 3a) and $t = 5 \cdot 10^{-5}$ s (Fig. 3b). The spatial graphs show the development of the strain process in time, caused by applying two concentrated forces (basic and control ones) to the plate. The Figures show that the normal displacement in the plate

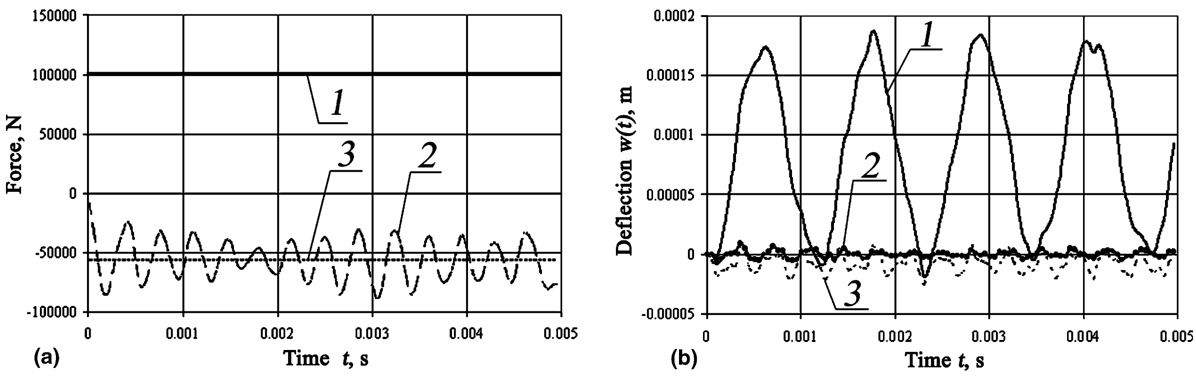

Fig. 2. Vibration control in the centre of the plate.

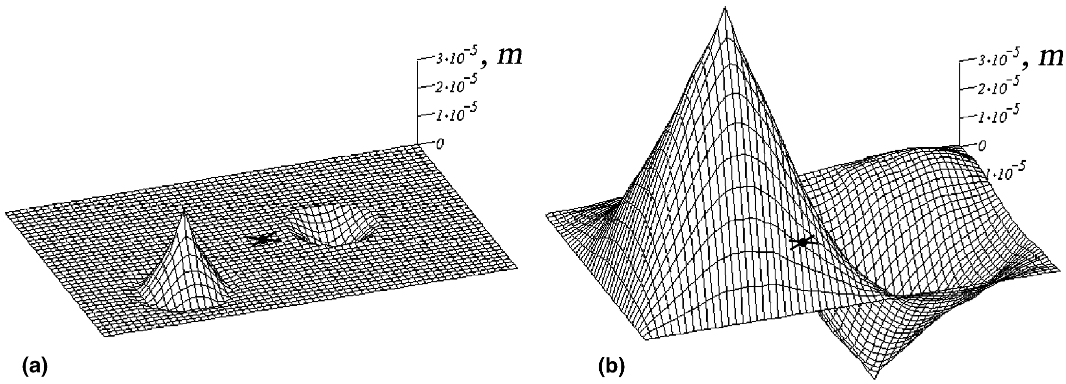

Fig. 3. Distribution of deflection over the plate median surface.

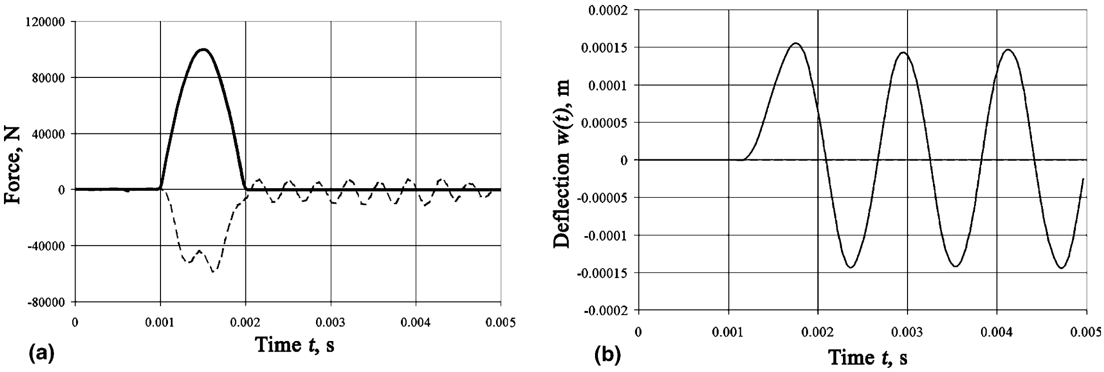

Fig. 4. Eliminating vibrations in a plate point.

centre (designated by a point in the Figure) is eliminated by superposition of the wave strain processes evoked by the basic and control forces.

We considered plate loading with a time-dependent force in the form of a sinusoidal half-wave. In so doing, absence of loading over a certain time interval (time interval from 0 to $1 \cdot 10^{-3}$ s in Fig. 4a) was taken into account. Fig. 4a shows the results of computing the control force, where the solid line on the graph corresponds to the law of disturbing force variation vs. time, and the dashed line represents the control force. Fig. 4b shows the deflections for the cases when there is no control (solid line), and when control is present (dashed line).

The technique of controlling the transverse vibrations of a plate point described can be simply generalized for other kinds of loads as well, e.g. for loads distributed over a rectangular or circular area. The two control schemes of those feasible for different kinds of loading are shown in Fig. 5.

In Fig. 5a, the basic load P is a concentrated force, whereas the control action G is a load uniformly distributed over the entire surface of the plate. In Fig. 5b, the basic load P is distributed uniformly over the entire surface of the plate, whereas the control action G is distributed over an area that is small as compared to plate dimensions. Vibration control is effected in an arbitrary point in the median plane of the plate.

The computation results for the scheme in Fig. 5a are given in Fig. 6. The description of Fig. 6 is similar to that of Fig. 4.

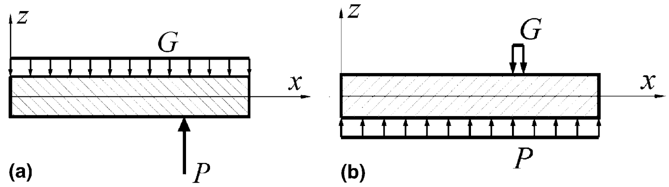

Fig. 5. Plate loading scheme.

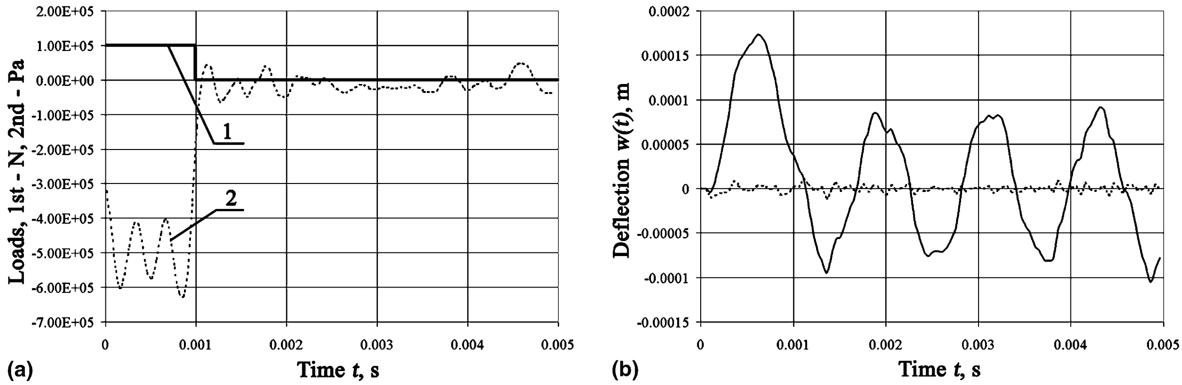

Fig. 6. Eliminating vibrations in a plate point.

5. Conclusion

The work presents the solution of the problem in controlling nonstationary strain processes in a rectangular plate whose equations of motion are accepted according to S.P. Timoshenko's theory. The possibility of controlling normal displacements in one point of the plate has been investigated on the assumption of action thereon of a concentrated impulse load (the basic one) and a concentrated nonstationary control load applied to a certain point of the plate. The sought-for function is the control load vs. time law of variation.

When solving the problems, we used the method of expanding the functions in the motions of equation into Fourier series ensuring an exact satisfaction of the boundary conditions along the plate contour. Based on the control criterion, the Volterra integral equation of the Ist kind was derived to determine the sought-for control load. The approximate solution of this equation was built by using Tikhonov's regularization method. The effectiveness of solving the problem stated has been demonstrated by several numeric cases.

Let us stress the importance of the problem described herein. By means of the method described in the paper, it is possible to obtain solutions for nonstationary-loaded elements of structures in the form of plates, which yield the given strain characteristics vs. time law of variation for an arbitrary point of these elements.

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